A generalization of the McKay conjecture for p-solvable groups

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joint work with HUIMIN CHANG

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- Furthermore, we establish a canonical bijection between these characters in the case where G has odd order. (We say that a set map is *canonical* if it is unambiguously defined and is independent of whatever choices may have been made in its construction.)
- Our proofs depend heavily on the theory of self-stabilizing pairs founded by M.L. Lewis, as well as some new results on π-special characters due to I.M. Isaacs and G. Navarro.

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I.M. Isaacs and G. Navarro, Characters of p'-degree of p-solvable groups, J. Algebra 246 (2001) 394-413.

M.L. Lewis, Obtaining nuclei from chains of normal subgroups, J. Alg. Appl. 5 (2006) 215-229.

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9. In this way, we have defined the $\mathcal{N}$-grading on $(\mathcal{N}, p)$-stable characters.

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(1) $\mathrm{Irr}_0(G,\mathcal{N}) = \mathrm{Irr}_{p'}(G)$ *and* $\mathrm{Irr}_0(\mathbf{N}_G(P),\mathcal{N}) = \mathrm{Irr}_{p'}(\mathbf{N}_G(P))$.

(2) $|\mathrm{Irr}_i(G,\mathcal{N})| = |\mathrm{Irr}_i(\mathbf{N}_G(P),\mathcal{N})|$ *for all $i \geq 0$, thus generalizing the McKay conjecture.*

(3) $\mathrm{Irr}(G,\mathcal{N}) = \bigsqcup_{i\geq 0} \mathrm{Irr}_i(G,\mathcal{N})$ *and* $\mathrm{Irr}(\mathbf{N}_G(P),\mathcal{N}) = \bigsqcup_{i\geq 0} \mathrm{Irr}_i(\mathbf{N}_G(P),\mathcal{N})$.

(4) $|\mathrm{Irr}(G,\mathcal{N})| = |\mathrm{Irr}(\mathbf{N}_G(P),\mathcal{N})|$.

(5) *For $i \geq 0$, $\mathrm{Irr}_i(G,\mathcal{N})$ is nonempty iff $p^i = \alpha(1)$ for some $\alpha \in \mathrm{Irr}(P,\mathcal{N})$.*

We note that Theorem A implies the strong form of the McKay conjecture for p-solvable groups, yet its proof is essentially the same one established by Isaacs and Navarro.

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I.M. Isaacs and G. Navarro, Characters of p'-degree of p-solvable groups, J. Algebra 246 (2001) 394-413.

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4. We can thus define p-special and p'-special characters of G.

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(4) *If $\alpha, \beta \in \mathrm{Irr}(P, \mathcal{N})$, then $\mathrm{Irr}_\alpha(G) = \mathrm{Irr}_\beta(G)$ iff α and β are $\mathbf{N}_G(P)$-conjugate, and otherwise, $\mathrm{Irr}_\alpha(G)$ and $\mathrm{Irr}_\beta(G)$ are disjoint.*

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The following is crucial for the proofs of Theorems A and B.

Theorem C (Chang-J)

Let $\Psi : \mathrm{Irr}(P) \to \mathrm{Char}(G)$ be the standard map defined as above. For any normal p-series $\mathcal{N}$ of G, the following then hold.

(1) *If $\alpha \in \mathrm{Irr}(P, \mathcal{N})$, then $\Psi(\alpha) \in \mathrm{Irr}(G)$.*

(2) *If $\alpha, \beta \in \mathrm{Irr}(P, \mathcal{N})$, then $\Psi(\alpha) = \Psi(\beta)$ iff α and β are $\mathbf{N}_G(P)$-conjugate.*

(3) *For each $\alpha \in \mathrm{Irr}(P, \mathcal{N})$, let W be the unique largest subgroup of G to which α extends and let γ be the unique p-special extension of α to W. Then $\delta \mapsto (\gamma\delta)^G$ defines an injection from the set of p'-special characters of W into $\mathrm{Irr}(G)$.*

Write $\mathrm{Irr}_\alpha(G)$ for the image of this injection, i.e.,

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Thank you for your attention!